MATH 2900: The 'Size' of Sets

DEFINITION: If X and Y are sets, we say X and Y are **equipotent** and write $X \sim Y$ iff there exists $F : X \rightarrowtail\!\!\!\!\!\twoheadrightarrow Y$.

EXAMPLE: Prove $X = \{a, b, c\}$ and $Y = \{1, 2, 3\}$ are equipotent.

EXAMPLE: Prove $X = \{a, b, c\}$ and $Y = \{1, 2\}$ are not equipotent.

EXAMPLE: Let $\mathbb{N} = \{1, 2, 3, \ldots\}$ and let $2\mathbb{N} = \{2, 4, 6, 8, \ldots\}$. Prove $\mathbb{N} \sim 2\mathbb{N}$.

EXAMPLE: Let $f(x) = \cot(\pi x)$. Show $f : (0, 1) \to \mathbb{R}$ is a bijection and hence $(0, 1) \sim \mathbb{R}$.

EXAMPLE: Suppose X and Y are sets. Prove:

- $X \sim X$. (i.e., '$\sim$' is **reflexive**.)

- If $X \sim Y$, then $Y \sim X$. (i.e., '$\sim$' is **symmetric**.)

- If $X \sim Y$ and $Y \sim Z$, then $X \sim Z$. (i.e., '$\sim$' is **transitive**.)

DEFINITION: A set X is called **finite** iff $X = \emptyset$ or $X \sim \{1, 2, \ldots n\}$ for some $n \in \mathbb{N}$.

A set X is called **infinite** iff X is not finite.

THEOREM: $\{1, 2, \ldots n\} \sim \{1, 2, \ldots m\}$ iff $n = m$.